Dimensional analysis application when calculating heat losses

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Abstract*—The article describes the procedure for the determination of heat loss through a mathematical model, which was processed through a dimensional analysis. The output of this model is a simple relationship where, based on the locating constant and the regression coefficient, it is possible to determine the loss of the piping system. Subsequently, this method was applied to calculate the heat loss of a pre-insulating pipe buried underground.*

***Keywords*—heat loss, heat network, model, dimensional analysis.**

I. INTRODUCTION

Currently, the determination of heat losses is most often achieved through the so-called balancing method, based on the experimental detection of temperature drops into and out of the network. In order to determine the total heat loss of a given section based on the balance method, it is necessary to know the temperature of the heat transfer medium at the beginning and end of the examined section with sufficient precision (at least to two decimal places for short sections) and the instantaneous mass flow rate of the water.

The determination of heat loss through this method is only suitable for very long sections of heat distribution, where the error in measuring the temperature difference does not cause a significant error in the following determination of heat losses. To measure the temperature difference of the heat transfer medium of the short sections, a top measuring technique is required, which cannot be permanently installed in each network, especially when the combined distribution system is several dozen kilometers long.

II. MATHEMATICAL MODEL OF HEAT LOSS

A dimensional analysis was used to design a new model to obtain information on the interconnection of relevant variables affecting heat loss through the creation of dimensionless arrays.

The advantage of a dimensional analysis is that it is possible to obtain a small number (two or three) from a large number of physical variables, on which the phenomenon depends, of a small number of non-dimensional arrays. The second advantage is that the acquired functional dependence has a universal validity for all other heat distribution meeting geometric and heat similarity with the examined distribution.

The basis of the mathematical model is a dimensional analysis, based on the principle of the dimensional homogeneity of the equations.

A dimensional analysis requires a preliminary analysis of the nature of the phenomenon. It serves to find the variables from a set of parameter variables affecting the examined phenomenon, and to compile a set of the minimum number of dimensionless arrays comprised from them.

When applying a dimensional analysis, the most important phase is the correct selection of transmuted variables that can affect the phenomenon.

During the research, we investigated the interrelationship of the involved transmuted variables, each of which can be dependent on others. The majority of all variables (except one) can be independently controlled. This one transmuted variable thus becomes a dependent variable. When selecting dependent variables, we often introduce a variable in the assembled relationship, which is not transmuted in the examined conditions, but comprise a dimensionless number in combination with the transmuted variables[2].

Including more than one transmuted variable in an examined relationship is a mistake as serious as omitting any of the involved variables affecting the examined phenomenon. On the contrary, it is not a problem to introduce variables into a relationship that does not have any effect on the phenomenon because it will arise from another solution. In order to correctly select the involved transmuted variables, it is necessary to create a certain idea of the examined phenomenon beforehand and to consider which independent variables must be taken into account in the description of the action. It results that from the examined phenomenon we must learn from experience or analogy why it can be influenced by a certain transmuted physical variable.

Two methods [2] are used to describe the phenomenon based on a dimensional analysis:

- The Rayleigh Method,
- The Buckingham Method, the so-called - theorem, which is used more frequently.

III. CREATION OF THE HEAT LOSS MODEL

To As mentioned above, the mathematical model for calculating the total heat loss (heat output) is based on a dimensional analysis and its mathematical interpretation is influenced by the correct choice of relevant variables by which the phenomenon is assumed to be significantly influenced.

Among the relevant variables affecting the heat losses of heat networks, based on the experience of the heat network operator and the professional literature, the quantities characterizing heat losses are easily measurable in practice. For the following quantities

- temperature of the heat transfer medium T_i (K)
- the ambient temperature T_e (K)
- the heat conductivity of the insulation λ_{iz} $(W \cdot m^{-1} \cdot K^{-1})$
- the heat conductivity of the soil λ_{zem} $(W \cdot m^{-1} \cdot K^{-1})$
- the depth of the pipe buried in the ground H (m)
- the mass flow water rate in the pipe Q_m $(m \cdot s^{-1})$
- specific heat loss (linear heat flow density) q_l $(W \cdot m^{-1})$
- theinsulation’s outside diameter d_3 (m)

The length dimension is repeated among the selected relevant variables. Only one physical variable of the same size group can be included in the solution. On this basis, it is possible to directly construct a dimensionless array called a simplex, whose shape is as follows:

$$\pi_1 = \frac{d_3}{H}, \qquad \text{or} \qquad \pi_1 = \frac{H}{d_3} \tag{1}$$

Based on the above, the complete physical equation expressing the dependence of relevant variables is the function of just the selected relevant variables and has the following form:

$$\varphi(q_l, T_i, H, \lambda_{zem}, Q_m) = 0. \tag{2}$$

Based on the dimensional diversity of the relevant variables, these will be displayed in groups, i.e.

$$\pi_i = q_l^{x_1} \cdot T_i^{x_2} . \cdot H^{x_3} \cdot \lambda_{zem}^{x_4} \cdot Q_m^{x_5} . \tag{3}$$

The dimension matrix A has $n = 5$ columns and $m = 4$ rows for the base units and is in the form of:

$$\begin{array}{c} \\ kg \\ m \\ s \\ K \end{array} \begin{array}{ccccc} q_l & T_i & H & \lambda_{zem} & Q_m \\ \end{array}$$

$$\begin{array}{c|ccccc|} & q_l & T_i & H & \lambda_{zem} & Q_m \\ \hline kg & 1 & 0 & 0 & 1 & 1 \\ m & 1 & 0 & 1 & 1 & 0 \\ s & -3 & 0 & 0 & -3 & -1 \\ K & 0 & 1 & 0 & -1 & 0 \end{array} . \tag{4}$$

With the value of matrix h = 4 and the number of relevant variables n = 5 it is possible to create the total of $i = n - h$, i.e. 1 dimensionless array π. The unknown variables cannot be uniquely determined because the number of unknown $x_i > m$. Therefore, the rectangular matrix is divided into two parts in terms of the dimensional analysis. The first part of the matrix will be square with the number of h columns and h rows (matrix P, see below), with the columns of the matrix selected so that the determinant is nonzero ($\Delta_P \neq 0$). It corresponds to the distribution of the x_i vector. The form of the square matrix **P** and the vector of the unknown variables x_i, indicated as **R**, is written in the simplified form:

$$\mathbf{P} \,.\, \mathbf{R} = (-1) \,.\, \mathbf{Q} \,.\mathbf{S} \,, \tag{5}$$

where Q - vector array with the number h = 1 and the number of rows n = 4,
S - vector of unknown variable with the number of h = 1 columns and the number of rows n = 1.

Equation (5) expressed through (3) and (4) can be represented in a broken-down form through the relationship

$$\begin{Vmatrix} 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & -3 & -1 \\ 1 & 0 & -1 & 0 \end{Vmatrix} \,.\, \begin{Vmatrix} x_2 \\ x_3 \\ x_4 \\ x_5 \end{Vmatrix} = (-1) \,.\, \begin{Vmatrix} 1 \\ 1 \\ -3 \\ 0 \end{Vmatrix} \,.\, \|x_1\| \tag{6}$$

The determinant of the matrix P is determined e.g. based on the Laplace development. We performed the choice of the excess unknown x_1, while both choices must be linearly independent. The matrix of choices has the form:

$$\begin{matrix} & x_1 & \\ & \begin{Vmatrix} 1 \\ 0 \end{Vmatrix} & \begin{matrix} 1^{st}\ choice \\ 2^{nd}\ choice \end{matrix} \end{matrix} \tag{7}$$

By multiplying the matrices according to (5), there applies

$$\|\mathbf{P}\|_{4\,x\,4} \,.\, \|\mathbf{R}\|_{4\,x\,1} = \|\mathbf{C}\|_{4\,x\,1}, \tag{8}$$

$$\|\mathbf{Q}\|_{4\,x\,2} \,.\, \|\mathbf{S}\|_{2\,x\,1} = \|\mathbf{F}\|_{4\,x\,1}. \tag{9}$$

Later it applies that

$$\|\boldsymbol{C}\|_{4\,x\,1} = (-1) \,.\, \|\boldsymbol{F}\|_{4\,x\,1}. \tag{10}$$

Since the type of matrices in expression (10) is the same, for the matrix elements from the equation (6) in the application of this relationship must apply:

$$x_4 + x_5 = -x_1$$

$$x_3 + x_4 = -x_1$$

$$-3x_4 - x_5 = 3x_1$$

$$x_2 - x_4 = 0$$

By solving this set of linear equations we attain an independent vector as output

$$\begin{matrix} & x_1 & x_2 & x_3 & x_4 & x_5 \\ \pi_2 & |\,1 & -1 & 0 & -1 & 0\,| \end{matrix} \tag{11}$$

This solution corresponds to the dimensionless array in the form:

$$\pi_2 = \frac{q_l}{T_i \,.\, \lambda_{zem}} \tag{12}$$

According to the rules of the dimensional analysis from relationship (11), it follows that the physical variable, which answers the unknown x_3 and x_5, does not affect heat loss, since its value is zero, meaning that it has arisen from the solution.

The dimensionless form of the heat loss function can be written in the form:

$$\Psi\ (\pi_1,\ \pi_2) = 0 \tag{13}$$

The dimensionless array π_2 contains the variable q_l , therefore this array can be expressed as a function of independent array π_1 in the form:

$$\pi_2 = \varphi\,(\pi_1) \tag{14}$$

The actual progress of the function of these two dimensionless arrays can be described e.g. power function in the form:

$$\pi_2 = \mathrm{A}\ .\ \pi_1^{\mathrm{B}}\ , \tag{15}$$

which in terms of further solution is suitably transformed into logarithmic coordinates to linear dependence

$$\log\pi_2 = \log \mathrm{A} + \mathrm{B} \cdot \log \pi_1 \tag{16}$$

By adding physical variables to dimensionless arrays in relationship (15), we reach the expression to determine the value of the total heat loss in the form:

$$\frac{q_l}{T_{\mathrm{i}}\ .\ \lambda_{\mathrm{zem}}} = A \cdot \left(\frac{d_3}{H}\right)^B \tag{17}$$

or $$q_l = A.\,T_{\mathrm{i}}.\,\lambda_{\mathrm{zem}}.\,d_3^{\,B}.\,H^{-B} \quad (\mathrm{W{\cdot}m^{-1}}) \tag{18}$$

The relationship (18) is universally applicable to all types of heat systems, both aboveground and underground, both inlet and return. To determine the specific value of a measured heat loss q_l according to relationship (18), for any type of the above-mentioned heat networks, it is necessary to respect the change in locating constant A and the regression coefficient B. It is possible for you to gain the type of network only on the basis of an experiment.

The characteristics of the heat network under detailed investigation are as follows: the network is underground, the nominal diameter DN125, the outside pipe diameter d_2 = 133 mm, the wall thickness of the pipe s = 3.6 mm, the insulation thickness s_{iz} = 33.5 mm, the length of the examined network L = 78 m, the mean value of the heat conductivity coefficient of the insulation (as a function of the water temperature in the inlet pipe) λ_{iz}= 0.041 $\mathrm{W{\cdot}m^{-1}{\cdot}K^{-1}}$, the mean value of the heat conductivity coefficient of the insulation) λ_{iz} = 0.040 $\mathrm{W{\cdot}m^{-1}{\cdot}K^{-1}}$, the mean value of the heat conductivity coefficient of soil λ_z = 1.35 $\mathrm{W{\cdot}m^{-1}{\cdot}K^{-1}}$, the mean value of the heat transfer coefficient from the surface of the earth to the external environment α_0 = 3 $\mathrm{W{\cdot}m^{-2}{\cdot}K^{-1}}$, the depth of the buried pipe H = 1.06 m.

The temperature of the water entering the inlet pipe ranged from 55 to 70 °C. Water temperature rising from the return pipe is from 50 to 60 °C. The average value of the ambient temperature t_e in the considered month was considered -1.5 °C (271.65 K).

The calculation of the locating constant A and the regression coefficient B in relationship (15) is performed on the basis of the smallest squares method. The second possibility to gain them is demonstrating the dependence of dimensionless π_2 and π_1 in the logarithmic coordinates directly. After selecting a regression line type and displaying the regression equation, it is possible to directly obtain information on the value of the locating constant and regression coefficient. The dependence of the dimensionless arrays π_2 and π_1 was obtained from a specific heat network conducted underground in channel less stored for the month January 2016 (Fig. 1 and 2)

It is the most suitable to determine A and B from the values obtained from the measurements of all the specified physical variables during the one year, especially for the inlet and for the return pipes. This will ensure the impact of a wide range of ambient temperature and the temperature of the heat transfer medium in the A and B values in functional dependence (15)

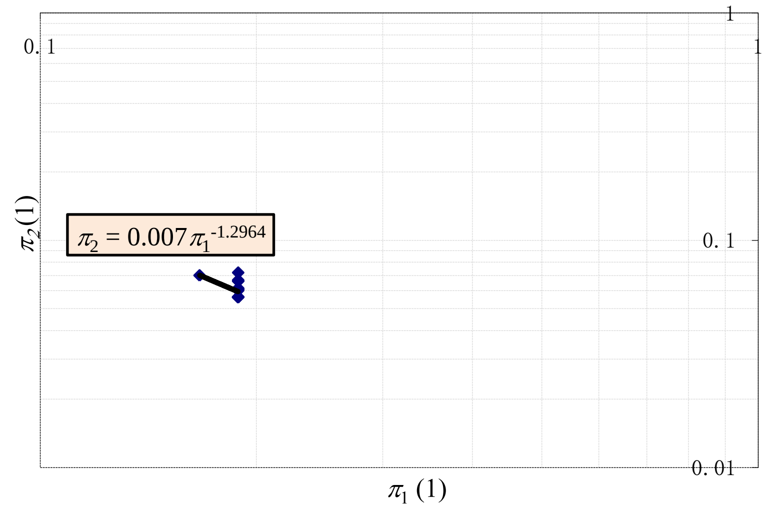

FIGURE 1: The calculation of the locating constant and the regression coefficient for the inlet pipe

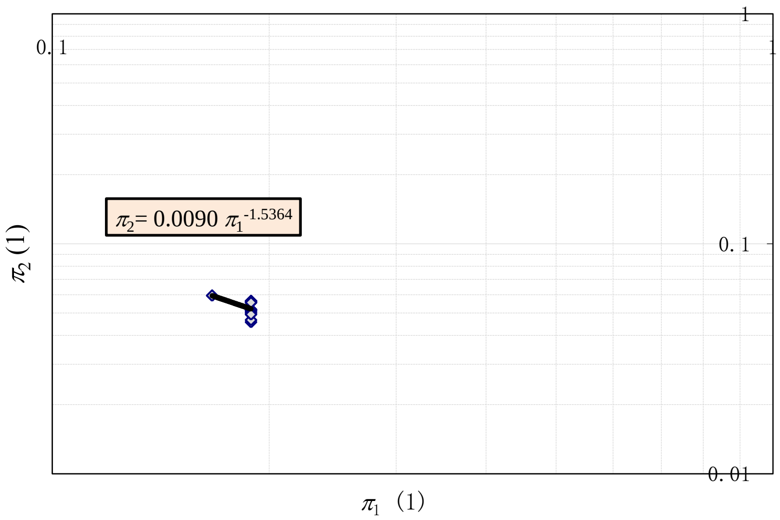

FIGURE 2: The calculation of the locating constant and the regression coefficient for the return pipe

Using Visual Basic, the locating constant, the regression coefficient and the relative heat loss according to (18) are calculated for the air temperature t_e from –20 to +30 ºC and the temperature of the heat transfer medium t_i from 45 to 70 ºC. Their progress is in Fig. 3 and 4.

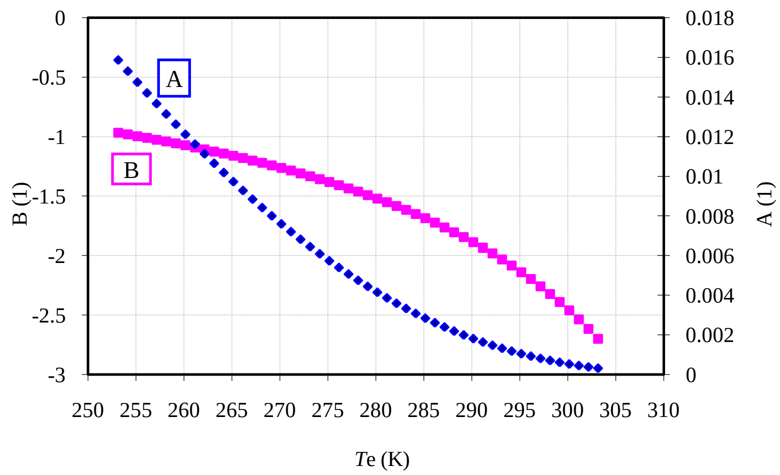

FIGURE 3: The progress of locating constant A and regression coefficient Bfor an ambient temperature range from –20 to + 30 ºC for the inlet pipe.

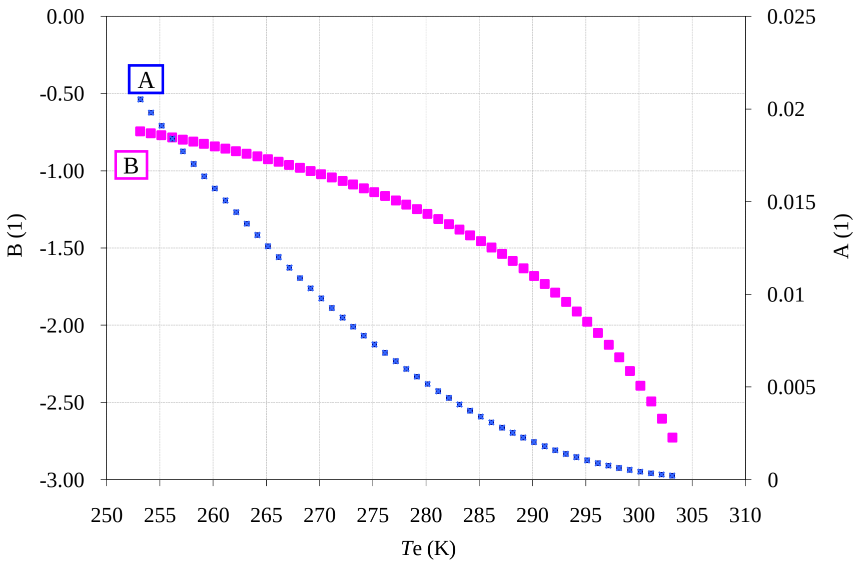

FIGURE 4: The progress of locating constant A and regression coefficient B for an ambient temperature range from –20 to + 30 ºC for the return pipe.

IV. CONCLUSION

The newly developed model for determining heat loss does not impose significant demands on the measurement of individual variables, it allows quick results on heat loss, it is reliable and offers some comfort. Putting it into actual practice would, from the point of view of both the manufacturer and the distributor of heat, does not signify any increase in finances and at the same time would make it possible to clearly determine the value of the total or specific heat loss of the network.

This model finds its justification mainly when examining loss on short pipe sections where recording of a drop in water temperature through regular thermometers is impossible or where little accurate information is obtained.

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