Electron-acoustic solitons and double layers in a plasma system with Tsallis distributed hot electrons

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Abstract— *A theoretical investigation is carried out for understanding the properties of electron acoustic solitary waves and double layers in an unmagnetized plasma whose constituents are cold electron fluid, hot electrons obeying Tsallis distribution and stationary ions. The Sagdeev potential approach is employed to carry out the analysis. The amplitude and nature of electron-acoustic solitary waves (EASWs) and electron-acoustic double layers (EADLs) are found to be extremely sensitive to the degree of nonextensivity. It is found that the present plasma model supports only rarefactive solitons and double layers depending on the q nonextensive parameter. The investigation shows that solitary structure ceases to exist when the parameter q crosses a certain limit.*

Keywords— Electron acoustic wave, Solitons, Double layer, Tsallis distribution.

I. INTRODUCTION

The electron acoustic (EA) wave is a high frequency wave that occurs in plasma with two distinct populations of electrons, referred as "cool" and "hot" [1]. Electron acoustic wave may also exist in electron-ion plasmas with ions hotter than electrons [2]. It is basically an electrostatic wave in which the inertia is provided by the cool electrons and the restoring force comes from the hot electrons. The ions play the role of a neutralizing background, i.e., the ion dynamics does not influence the EA waves because the EA wave frequency is much higher than the ion plasma frequency. The EA wave phase speed must be intermediate between the cold and hot electron thermal speeds such that the wave avoids damping by both the cold and the hot electron species. Electron acoustic waves often occur in laboratory plasmas [3-5] and space plasmas, e.g., in the Earth's bow shock [6-8], in the auroral magnetosphere [9,10] and in geomagnetic tail [11]. In the case of Earth's bow shock, heliospheric termination shock and planetary and neutron star magnetospheres, in addition to two electron populations, the presence of magnetic field-aligned electron beam is considered to be the main energy source for the excitation of this mode.

The electron and ion distributions play crucial role in characterizing the physics of nonlinear wave structures by increasing richness and variety of the wave motion in plasma. Also, they significantly influence the conditions required for the formation of solitons and double layers. Moreover, it is also known that electrons and ion distribution can be significantly modified in the presence of large amplitude waves. Over past few years, new statistical approach nonextensive statistics or Tsallis statistics has attracted much attention [12]. A suitable nonextensive generalization of the Boltzmann-Gibbs-Shannon (BGS) entropy for statistical equilibrium was first recognized by Renyi [13] and subsequently proposed by Tsallis [12] suitably extending the standard additivity of the entropies to the nonlinear, nonextensive case where one particular parameter, the entropic index q, characterizes the degree of nonextensivity of the considered system. The Maxwellian distribution in Boltzmann-Gibbs (BG) statistics is believed valid universally for the macroscopic ergodic equilibrium systems. But this concept fails to explain some phenomena which have complex behaviours such as long range interactions such as plasma and gravitational systems. Spacecraft measurements of plasma velocity distributions, both in the solar wind and in planetary magnetospheres and magnetosheaths, have revealed that non-Maxwellian distributions are quite common. Tsallis distribution is thought to be useful generalization of BG statistics and to be appropriate for the statistical description of the long-range interaction systems, characterizing the non-equilibrium stationary state [14-16]. In Tsallis distribution, q is the nonextensive parameter and for $q \neq 1$, it gives power law distribution functions. For $q < 1$, high energy states are more probable than in the extensive case. The distribution corresponds to power law behaviour for large energy. On the other hand, for $q > 1$ high energy states are less probable than in the extensive case, there is a cutoff beyond which no states exist. Boltzmann-Gibbs entropy is obtained from Tsallis entropy S_q if the parameter $q \rightarrow 1$ [12].

Malfliet and Wieers [17] reviewed the studies of solitary waves in plasma and found that Reductive Perturbation Technique (RPT) is based on the assumption of smallness of amplitude and so this technique can explain only small amplitude solitary waves. But there are situations where the excitation mechanism gives rise to large amplitude waves; to study such situation one should employ a non perturbative technique. Sagdeev potential method [18] is one such method to obtain solitary wave solutions. In this paper, the dynamics of solitons and double layers is studied in plasma system constituting of two temperature electrons with hot electrons obeying nonextensive distribution. Regarding the organization of the paper, basic equations of theoretical model and derivation of Sagdeev potential associated to solitons and double layers is presented in section II and section III is devoted to discussion of numerical results. The summary of conclusions drawn from work is given in section IV.

II. BASIC EQUATIONS

In the present model, unmagnetized plasma consisting of cold electron fluid, hot electrons obeying a nonextensive distribution and stationary ions are considered. The set of normalized fluid equations governing the dynamics of electron acoustic waves in such plasma are [19]:

$$\frac{\partial n_c}{\partial t} + \frac{\partial}{\partial x}(n_c u_c) = 0 \tag{1}$$

$$\frac{\partial u_c}{\partial t} + u_c \frac{\partial u_c}{\partial x} = \frac{\partial \phi}{\partial x} - \frac{\sigma}{n_c}\frac{\partial p_c}{\partial x} \tag{2}$$

$$\frac{\partial p_c}{\partial t} + u_c \frac{\partial p_c}{\partial x} + 3p_c \frac{\partial u_c}{\partial x} = 0 \tag{3}$$

$$\frac{\partial^2 \phi}{\partial x^2} = n_c + \beta n_h - (1+\beta) \tag{4}$$

Where $n_h = [1 + (q-1)\phi]^{q+1/2(q-1)}$, here q stands for strength of nonextensivity.

Here (1)-(4) are equations of continuity, motion, pressure and Poisson's equation respectively. In the above equations, $\sigma = \frac{T_c}{T_h}$ and $\beta = \frac{n_{h0}}{n_{c0}}$. p_c, n_c and n_h are the normalized pressure of cold electrons and the normalized number densities of cold and hot electrons, respectively. ϕ is the normalized electrostatic potential, u_c is normalized velocity of cold electrons. The densities of cold and hot electrons are normalized by n_{c0} and n_{h0}, respectively. The space coordinate *x*, time *t*, velocity, pressure and electrostatic potential ϕ are normalized by the hot electron Debye length $\lambda_{Dh} = \sqrt{\frac{K_B T_h}{4\pi n_{h0} e^2}}$, inverse of cold electron plasma frequency $\omega_{pc}{}^{-1} = \sqrt{\frac{m}{4\pi n_{c0} e^2}}$, the cold electron fluid speed $c_e = \sqrt{\frac{K_B T_h}{\alpha m}}$, $n_{c0} K_B T_c$ and $\frac{K_B T_h}{e}$, respectively.

To obtain a travelling wave solution, all the dependent variables depend on single independent variable $\xi = x - Mt$, where M is the Mach number, i.e., the velocity of the solitary wave. Further solving and substituting expressions into the Poisson's equation and assuming appropriate boundary conditions for the localized disturbance along with the conditions that potential ϕ, and $\frac{d\phi}{d\xi} = 0$ at $\xi \to \pm\infty$. This leads to the following energy integral

$$\frac{1}{2}\left(\frac{d\phi}{d\xi}\right)^2 + V(\phi) = 0 \tag{5}$$

where $V(\phi)$ is Sagdeev potential [18] and reads as

$$V(\phi) = \frac{1}{6\sqrt{3\sigma}}\left[\left(M+\sqrt{3\sigma}\right)^3 - \left(\mathrm{M}-\sqrt{3\sigma}\right)^3 - \left[2\phi + \left(M+\sqrt{3\sigma}\right)^2\right]^{3/2} + \left[2\phi + \left(M-\sqrt{3\sigma}\right)^2\right]^{3/2}\right] + \frac{2\beta}{3q-1}\left[1 - [1+(q-1)\phi]^{\frac{3q-1}{2(q-1)}}\right] + (1+\beta)\phi \tag{6}$$

Equation (5) yields solitary wave solutions when the Sagdeev potential $V(\phi)$ satisfies the following conditions:

i. $V(\phi) = 0$ and $\frac{dV(\phi)}{d\phi} = 0$ at $\phi = 0$.

ii. $\left(\frac{d^2V(\phi)}{d\phi^2}\right) < 0$ at $\phi = 0$.

iii. $V(\phi_m) = 0$ at $\phi = \phi_m$

iv. $V(\phi) < 0$ when $0 < \phi < \phi_m$ for positive solitary waves or $\phi_m < \phi < 0$ for negative solitary waves, where ϕ_m is the maximum potential.

Equation (6) satisfies condition (i) as $V(\phi)$ and its first derivative with respect to ϕ vanish at $\phi = 0$. The condition (ii) is satisfied provided $M > M_{min}$, where M_{min} is the lower limit of the Mach number, above which the solitary waves may be excited and is given by

$$M_{min} = \sqrt{3\sigma + \frac{2}{\beta(q+1)}} \quad (7)$$

III. RESULTS AND DISCUSSION

Sagdeev potential V (ϕ) given by equation (6) as a function of ϕ is numerically analyzed under various situations. For large amplitude solitary waves, Sagdeev potential V(ϕ) is plotted against electrostatic potential ϕ for different values of parameters. In present model, effect nonextensive parameter q on solitons and double layers is analyzed for three different ranges ($-1 < q < 0$, $0 < q < 1$ and $1 < q < 2$).

3.1 ELECTRON ACOUSTIC SOLITONS

The electrostatic potential exhibits spatially localized structures as is evident from well structure of Sagdeev potential. The electron acoustic solitary waves (EASWs) are obtained for all ranges of nonextensive parameter q and for all ranges of q only rarefactive solitary waves exist. In Fig. 1, Sagdeev potential V (ϕ) is plotted against the electrostatic potential ϕ for different values of q for the range $-1 < q < 0$. It is clear that as q increases ($q \rightarrow 0$), the potential pulse amplitude increases while width decreases: an increase in q makes the rarefactive solitary structure more spiky. There exists a critical value of q i.e. q_c below which solitons do not exist and $q_c = -0.8$ for chosen set of parameters. Fig. 2 shows the variation of Sagdeev potential V (ϕ) with ϕ for different values of Mach number M. It can be seen that with increase in Mach number, the faster moving pulses will be taller and narrower which is opposite for the case of slower ones which is in agreement with soliton phenomenology [20]. This trend of variation is same for other ranges of q ($0 < q < 1$ and $1 < q < 2$).

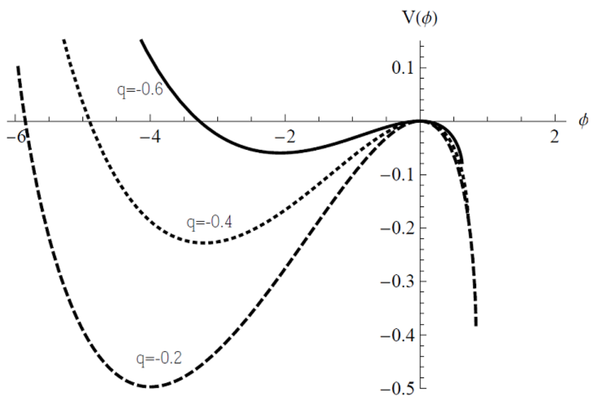

FIGURE 1. VARIATION OF SAGDEEV POTENTIAL V (ϕ) with ϕ for range $-1 < q < 0$ wth $\beta = 1$, $\sigma = 0.1$ and $M = 4$.

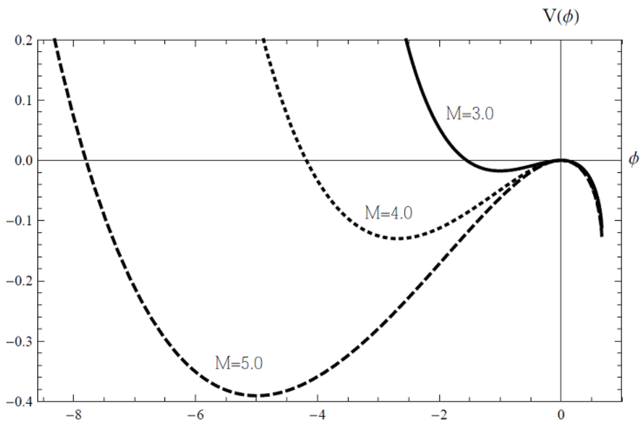

FIGURE 2. SAGDEEV POTENTIAL V(ϕ) vs ϕ FOR DIFFERENT VALUES OF M = 3, 4 and 5. THE OTHER PARAMETERS ARE $q = -0.5$, $\beta = 1$, $\sigma = 0.1$.

Fig. 3 shows the existence of rarefactive solitons for different values of q in range $0 < q < 1$. With increase of the nonextensive parameter q ($q \rightarrow 1$), the potential pulse amplitude increases while its width narrows. One can conclude that the electron acoustic solitons may become less smooth as electrons evolve toward their Maxwell-Boltzmann thermodynamic equilibrium ($q \rightarrow 1$).

Fig. 4 shows the Sagdeev potential for different values of q in range $1 < q < 2$. For different values of q, similar behaviour is observed as that for range $-1 < q < 1$, i.e., the existence of rarefactive solitons. Electron acoustic solitons exist for all values of q in range $1 < q < 2$.

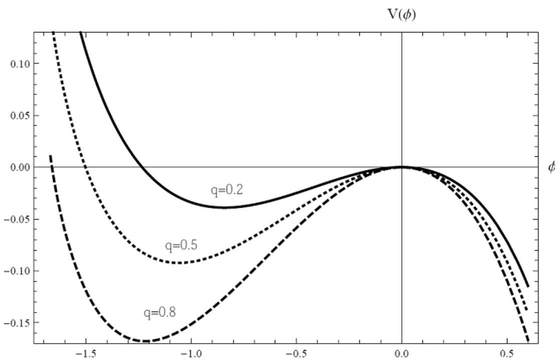

FIGURE 3. VARIATION OF SAGDEEV POTENTIAL V (ϕ) with ϕ for range 0 < q < 1 with β = 1, σ = 0.01 and M = 2.

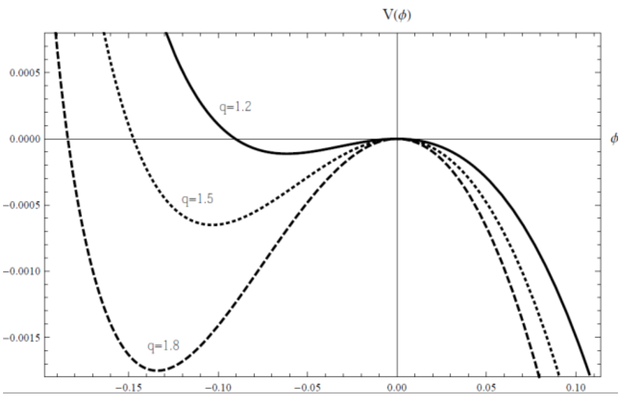

FIGURE 4. VARIATION OF SAGDEEV POTENTIAL V (ϕ) with ϕ for range 1 < q < 2 with β = 1, σ = 0.1 and M = 1.17.

3.2 ELECTRON ACOUSTIC DOUBLE LAYERS

In the small amplitude limit $\phi \ll 1$, equation (6) can be written as

$$V(\phi) = A_1\phi^2 + A_2\phi^3 + A_3\phi^4 + 0(\phi^5) \tag{8}$$

Where $A_1 = -\frac{\beta(q+1)}{4} + \frac{1}{6\sqrt{3\sigma}}\left[\frac{3}{2\sqrt{M^2-2M\sqrt{3\sigma}+3\sigma}} - \frac{3}{2\sqrt{M^2+2M\sqrt{3\sigma}+3\sigma}}\right]$,

$$A_2 = \frac{\beta(q-3)(q+1)}{24} + \frac{1}{6\sqrt{3\sigma}}\left[-\frac{1}{2(M^2-2M\sqrt{3\sigma}+3\sigma)^{3/2}} + \frac{1}{2(M^2+2M\sqrt{3\sigma}+3\sigma)^{3/2}}\right]$$

$$A_3 = -\frac{\beta(q-3)(q+1)(3q-5)}{192} + \frac{1}{6\sqrt{3\sigma}}\left[\frac{3}{8\left(M^2-2M\sqrt{3\sigma}+3\sigma\right)^{5/2}} - \frac{3}{8\left(M^2+2M\sqrt{3\sigma}+3\sigma\right)^{5/2}}\right]$$

Again for double layer solutions the following conditions must be satisfied:

v. $V(\phi = 0) = 0$ and $V(\phi = \phi_m) = 0$.

vi. $V'(\phi = 0) = 0$ and $V'(\phi = \phi_m) = 0$.

vii. $V''(\phi = 0) < 0$ and $V''(\phi = \phi_m) < 0$.

Conditions (v) and (vi) gives $2\phi_m = -A_2/A_3$, and Sagdeev potential $V(\phi)$ is

$$V(\phi) = A_3\phi^2(\phi_m - \phi)^2 \tag{9}$$

The double layer solution can be obtained as

$$\phi = \frac{\phi_m}{2}\left[1 - tanh\left(\frac{2\xi}{\Delta}\right)\right] \tag{10}$$

where $\Delta = \sqrt{-8/A_3}/|\phi_m|$ represents width of double layer provided $A_3 < 0$.

$$\text{i.e., } A_3 = -\frac{\beta(q-3)(q+1)(3q-5)}{192} + \frac{1}{6\sqrt{3\sigma}}\left[\frac{3}{8\left(M^2-2M\sqrt{3\sigma}+3\sigma\right)^{5/2}} - \frac{3}{8\left(M^2+2M\sqrt{3\sigma}+3\sigma\right)^{5/2}}\right] < 0$$

It is to be noted from (9) that nature of the double layer depends on the sign of A_2 i.e., for $A_2 > 0$ a compressive electron acoustic double layer exists (EADLs), whereas for $A_2 < 0$ we would have rarefactive electron acoustic double layer. By virtue of $A_2 > 0$, the compressive electron acoustic double layer requires that q > 3. For three ranges of q, only rarefactive electron acoustic double layers (EADLs) are found to exist.

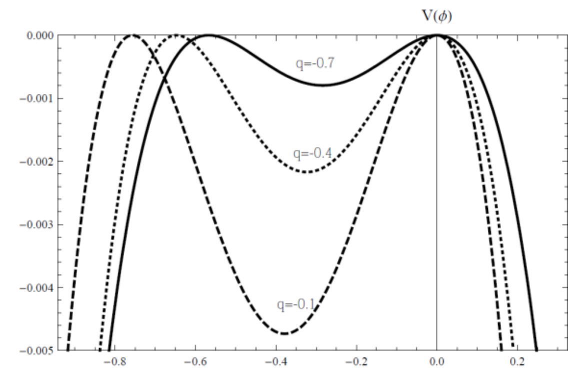

FIGURE 5 (A). VARIATION OF SAGDEEV POTENTIAL V(ϕ) with electrostatic potential ϕ for range -1 < q < 0. Here β = 3, σ = 0.1 and M = 5.

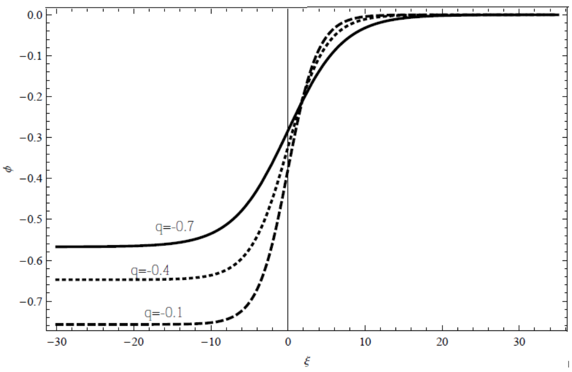

FIGURE 5 (B). DOUBLE LAYER SOLUTION FOR RANGE -1 < q < 0. Here β = 3, σ = 0.1 and M = 5.

Fig. 5 (a) and Fig. 5(b) show the effect of nonextensive parameter q on double layer structures. It can be seen that double layer excitations are amplified as q increases. Increase in q makes the Sagdeev potential well deeper. Increase in Sagdeev potential well depth means that the amplitude increases, width decreases and vice versa. In figure 5 (b), increase in q increases the width of rarefactive electron acoustic double layer. It may be mentioned here that for the range 0<q<1,

Again negative double layers are reported and the results observed are similar to those observed for the range $-1 < q < 0$. The range $q > 1$ provides qualitatively the same result as before as the electrons evolve far away from their thermodynamic equilibrium ($q > 1$), potential pulse becomes spikier and the rarefactive electron acoustic double layer width increases.

IV. CONCLUSION

In this paper, electron acoustic solitary waves and double layers have been studied in an unmagnetized plasma consisting of cold electrons, nonextensive hot electrons and stationary ions. The Sagdeev potential method is used to investigate large amplitude localized solitons and double layers. Exact Sagdeev potential is derived in presence of nonextensive hot electrons and studied numerically to see the effect of nonextensive parameter q on existence of solitons and double layers. The present plasma model supports only rarefactive EASWs and EADLs. When q reaches certain limit, solitons and double layers cease to exist. Numerical results reveal that the width and amplitude of electron acoustic solitons and double layers are affected significantly because of the nonextensive nature of hot electrons. The results are in agreement with Danehkar [19], Sahu [21] and Pakzad [22]. Danehkar [19] and Sahu [21] considered the superthermal distribution of hot electrons and Pakzad [22] carried out the analysis with the Reductive Perturbation Technique (RPT).

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